83. D_s^+ Branching Fractions

Updated November 2015 by J.L. Rosner (University of Chicago) and C.G. Wohl (LBNL).

(Note added April 2018. There have been few measurements of D_s^+ branching ratios since our 2015 revision, and none at all of hadronic modes. So we run with the 2015 version.)

Figure 83.1 shows a partial breakdown of the D_s^+ branching fractions. The rest of this note is about how the figure was constructed. The values shown make heavy use of CLEO measurements of inclusive branching fractions [1]. For references to other data cited in the following, see the Listings.

83.1. Modes with leptons

The bottom $(19.9 \pm 0.9)\%$ of Fig. 83.1 shows the fractions for the modes that include leptons. Measured $Xe^+\nu_e$ semileptonic fractions have been doubled to include the $X\mu^+\nu_\mu$ fractions. The sum of the exclusive $Xe^+\nu_e$ fractions is $(6.9 \pm 0.4)\%$, consistent with an inclusive semileptonic measurement of $(6.5 \pm 0.4)\%$. There seems to be little missing here.

83.2. Inclusive hadronic $K\overline{K}$ fractions

The Cabibbo-favored $c \to s$ decay in D_s^+ decay produces a final state with both an s and an $\bar{s}$; and thus modes with a $K\overline{K}$ pair or with an η, ω, η', or ϕ predominate (as may already be seen in Fig. 83.1 in the semileptonic fractions). We consider the $K\overline{K}$ modes first. A complete picture of the exclusive $K\overline{K}$ charge modes is not yet possible, because branching fractions for many of those modes have not yet been measured. However, CLEO has measured the inclusive K^+, K^-, K_S^0, K^+K^-, $K^+K_S^0$, $K^-K_S^0$, and $2K_S^0$ fractions (these include modes with leptons) [1]. And each of these inclusive fractions with a K_S^0 is equal to the corresponding fraction with a K_L^0: $f(K^+K_L^0) = f(K^+K_S^0)$, $f(2K_L^0) = f(2K_S^0)$, *etc.* Therefore, of all inclusive fractions pairing a K^+, K_S^0, or K_L^0 with a K^-, K_S^0, or K_L^0, we know all but $f(K_S^0 K_L^0)$.

We can get that fraction. The total K_S^0 fraction is

$$f(K_S^0) = f(K^+K_S^0) + f(K^-K_S^0) + 2f(2K_S^0) + f(K_S^0K_L^0) \\ + f(\text{single } K_S^0) \ ,$$

where $f(\text{single } K_S^0)$ is the sum of the branching fractions for modes such as $K_S^0\pi^+2\pi^0$ with a K_S^0 and no second K. The $K_S^0\pi^+2\pi^0$ mode is in fact the only unmeasured single-K_S^0 mode (throughout, we shall assume that fractions for modes with a K or $K\overline{K}$ and more than three pions are negligible), and we shall take its fraction to be the same as for the $K_S^0 2\pi^+\pi^-$ mode, $(0.30 \pm 0.11)\%$. Any reasonable deviation from this value would be too small to matter much in the following. Adding the several small single-K_S^0 branching fractions, including those from semileptonic modes, we get $f(\text{single } K_S^0) = (1.65 \pm 0.26)\%$.

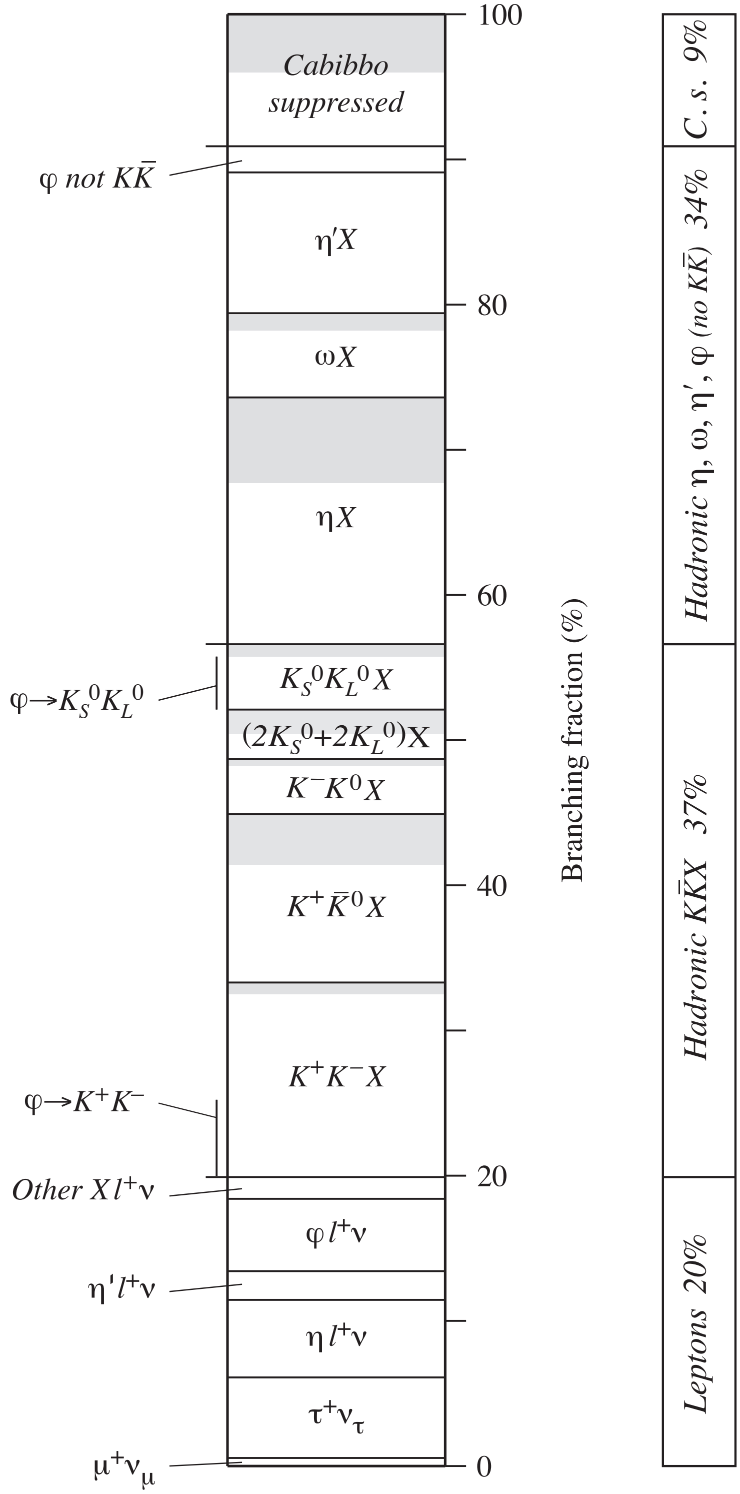

Figure 83.1: A partial breakdown of D_s^+ branching fractions. The hadronic bins in the left column show inclusive fractions. Shading within a bin shows how much of the inclusive fraction is not yet accounted for by adding up all the relevant exclusive fractions. The inclusive hadronic ϕ fraction is spread over three bins, in proportion to its decay fractions into K^+K^-, $K_S^0K_L^0$, and no-$K\bar{K}$ modes.

Using this, we have:

$$\begin{aligned} f(K_S^0 K_L^0) &= f(K_S^0) - f(K^+ K_S^0) - f(K^- K_S^0) \\ &\quad - 2f(2K_S^0) - f(\text{single } K_S^0) \\ &= (19.0 \pm 1.1) - (5.8 \pm 0.5) - (1.9 \pm 0.4) \\ &\quad - 2 \times (1.7 \pm 0.3) - (1.7 \pm 0.3) \\ &= (6.2 \pm 1.4)\% \ . \end{aligned}$$

Here and below we treat the errors as uncorrelated, although often they are not. However, our main aim is to get numbers for Fig. 83.1; errors are secondary.

There is a check on our result: The ϕ inclusive branching fraction is $(15.7 \pm 1.0)\%$, of which 34%, or $(5.34 \pm 0.34)\%$ of D_s^+ decays, produces a $K_S^0 K_L^0$. Our $f(K_S^0 K_L^0) = (6.2 \pm 1.4)\%$ has to be at least this large—and it is.

We now have all the inclusive $K\overline{K}$ fractions. We use $f(K^+\overline{K}^0) = 2\ f(K^+K_S^0)$, and likewise for $f(K^-K^0)$. For K^+K^- and $K_S^0K_L^0$, we subtract off the contributions from $\phi\ell^+\nu$ decay to get the purely hadronic $K\overline{K}$ inclusive fractions:

$$\begin{aligned} f(K^+K^-, \text{hadronic}) &= (15.8 \pm 0.7) - (2.44 \pm 0.14) \\ &= (13.4 \pm 0.7)\% \\ f(K^+\overline{K}^0, \text{hadronic}) &= (11.6 \pm 1.0)\% \\ f(K^-K^0, \text{hadronic}) &= (3.8 \pm 0.8)\% \\ f(2K_S^0 + 2K_L^0, \text{hadronic}) &= (3.4 \pm 0.64)\% \\ f(K_S^0K_L^0, \text{hadronic}) &= (6.2 \pm 1.4) - (1.70 \pm 0.10) \\ &= (4.5 \pm 1.4)\% \ . \end{aligned}$$

The fractions are shown in Fig. 83.1. They total $(36.7 \pm 2.1)\%$ of D_s^+ decays.

We can add more information to the figure by summing up measured branching fractions for exclusive modes within each bin:

K^+K^- *modes*—The sum of measured $K^+K^-\pi^+$, $K^+K^-\pi^+\pi^0$, and $K^+K^-2\pi^+\pi^-$ branching fractions is $(12.6 \pm 0.6)\%$. That leaves $(0.8 \pm 0.9)\%$ for the $K^+K^-\pi^+2\pi^0$ mode, which is the only other K^+K^- mode with three or fewer pions. In Fig. 83.1, this unmeasured part of the K^+K^- bin is shaded.

$K^+\overline{K}^0$ *modes*—Two times the sum of the measured $K^+K_S^0$, $K^+K_S^0\pi^0$, and $K^+K_S^0\pi^+\pi^-$ branching fractions is $(8.1 \pm 0.5)\%$. This leaves $(3.5 \pm 1.1)\%$ for the unmeasured $K^+\overline{K}^0$ modes (there are three such modes with three or fewer pions). This is shaded in the figure.

K^-K^0 *modes*—Twice the $K^-K_S^02\pi^+$ fraction is $(3.34 \pm 0.20)\%$, which leaves about $(0.5 \pm 0.8)\%$ for $K^-K^02\pi^+\pi^0$, the only other K^-K^0 mode with three or fewer pions.

$2K_S^0 + 2K_L^0$ *modes*—The $2K_S^0\pi^+$ and $2K_S^0 2\pi^+\pi^-$ fractions sum to $(0.86 \pm 0.07)\%$; this times two (for the corresponding $2K_L^0$ modes) is $(1.72 \pm 0.14)\%$. This leaves about $(1.7 \pm 0.7)\%$ for other $2K_S^0 + 2K_L^0$ modes.

$K_S^0 K_L^0$ *modes*—Most of the $K_S^0 K_L^0$ fraction is accounted for by ϕ decays (see below).

83.3. Inclusive hadronic η, ω, η', and ϕ fractions

These are easier. We start with the inclusive branching fractions, and then, to avoid double counting, subtract: (1) fractions for modes with leptons; (2) η mesons that are included in the inclusive η' fraction; and (3) K^+K^- and $K_S^0 K_L^0$ from ϕ decays:

$$\begin{aligned}
f(\eta \text{ hadronic}) &= f(\eta \text{ inclusive}) - 0.65\, f(\eta' \text{ inclusive}) \\
-f(\eta\ell^+\nu) &= (17.0 \pm 3.1)\% \\
f(\omega \text{ hadronic}) &= f(\omega \text{ inclusive}) - 0.0275\, f(\eta' \text{ inclusive}) \\
&= (5.8 \pm 1.4)\% \\
f(\eta' \text{ hadronic}) &= f(\eta' \text{ inclusive}) - f(\eta'\ell^+\nu) \\
&= (9.7 \pm 1.9)\% \\
f(\phi \text{ hadronic}, \not\to K\overline{K}) &= 0.17\, \left[f(\phi \text{ inclusive}) - f(\phi\ell^+\nu)\right] \\
&= (1.8 \pm 0.2)\% \ .
\end{aligned}$$

The factors 0.65, 0.0275, and 0.17 are the $\eta' \to \eta$, $\eta' \to \omega$, and $\phi \not\to K\overline{K}$ branching fractions. Figure 83.1 shows the results; the sum is $(34.2 \pm 3.9)\%$, which is about equal to the hadronic $K\overline{K}$ total.

Note that the bin marked ϕ near the top of Fig. 83.1 includes neither the $\phi\ell^+\nu$ decays nor the 83% of other ϕ decays that produce a $K\overline{K}$ pair. There is twice as much ϕ in the $K_S^0 K_L^0$ bin, and nearly three times as much in the K^+K^- bin. These contributions are indicated in those bins.

Again, we can show how much of each bin is accounted for by measured exclusive branching fractions:

η *modes*—The sum of $\eta\pi^+$, $\eta\rho^+$, and ηK^+ branching fractions is $(11.1 \pm 1.2)\%$, which leaves a good part of the inclusive hadronic η fraction, $(17.0 \pm 3.1)\%$, to be accounted for. This is shaded in the figure.

ω *modes*—The sum of $\omega\pi^+$, $\omega\pi^+\pi^0$, and $\omega 2\pi^+\pi^-$ fractions is $(4.6 \pm 0.9)\%$, which is nearly as large as the inclusive hadronic ω fraction, $(5.8 \pm 1.4)\%$.

η' *modes*—The sum of $\eta'\pi^+$, $\eta'\rho^+$, and $\eta' K^+$ fractions is $(9.7 \pm 1.9)\%$, which agrees with the inclusive hadronic η' fraction, $(9.7 \pm 1.9)\%$. (An old measurement of the $\eta'\rho^+$ fraction, $(12.5 \pm 2.2)\%$, has been abandoned [2].)

83.4. Cabibbo-suppressed modes

The sum of the fractions for modes with a $K\bar{K}$, η, ω, η', or leptons is $(90.8 \pm 4.5)\%$. The remaining $(9.2 \pm 4.5)\%$ is to Cabibbo-suppressed modes, mainly single-$K+$ pions and multiple-pion modes (see below). However, it should be noted that some small parts of the modes already discussed are Cabibbo-suppressed. For example, the $(1.10 \pm 0.24)\%$ of D_s^+ decays to $K^0 \ell\nu$ or $K^{*0}\ell\nu$ is already in the $X\ell\nu$ bin in Fig. 83.1. And the inclusive measurements of η, ω, and η' fractions do not distinguish between (and therefore include both) Cabibbo-allowed and -suppressed modes. We shall not try to make a separation here.

K^0 + pions—Above, we found that $f(\text{single } K_S^0) = (1.65 \pm 0.26)\%$. Subtracting leptonic fractions with a K_S^0 leaves $(1.22 \pm 0.28)\%$. The hadronic single-K^0 fraction is twice this, $(2.44 \pm 0.56)\%$. The sum of measured $K^0\pi^+$, $K^0\pi^+\pi^0$, and $K^0 2\pi^+\pi^-$ fractions is $(1.84 \pm 0.28)\%$.

K^+ + pions—The $K^+\pi^0$ and $K^+\pi^+\pi^-$ fractions sum to $(0.72 \pm 0.05)\%$. Much of the $K^+ n\pi$ modes, where $n \geq 3$, is already in the η, ω, and η' bins, and the rest is not measured. The total K^+ fraction wanted here is probably in the 1-to-2% range.

Multi-pions—The $2\pi^+\pi^-$, $\pi^+ 2\pi^0$, and $3\pi^+ 2\pi^-$ fractions total $(2.54 \pm 0.16)\%$. Modes not measured might double this.

The sum of the actually measured fractions is $(5.1 \pm 0.3)\%$, which is not inconsistent with the Cabibbo-suppressed total of $(9.2 \pm 4.5)\%$.

83.5. A model

With CLEO about to publish inclusive branching fractions [1], Gronau and Rosner predicted those fractions using a "statistical isospin"model [3]. Consider, say, the $D_s^+ \to K\overline{K}\pi$ charge modes: the $K^+K^-\pi^+$ branching fraction is measured, the $K^+\overline{K}^0\pi^0$ and $K^0\overline{K}^0\pi^+$ fractions are not. The statistical isospin model assumes that all the independent isospin amplitudes for $D_s^+ \to K\overline{K}\pi$ decay are equal in magnitude and incoherent in phase—in which case, the ratio of the three fractions here is 3:3:2. (Actually, use was also made of the fact that $D_s^+ \to K\overline{K}\pi$ decay is dominated by $\phi\pi^+$, $K^+\overline{K}^{*0}$, and $K^{*+}\overline{K}^0$ submodes; but the estimated charge-mode ratios were not far from 3:3:2.) A different, quark-antiquark pair-production model was used to estimate systematic uncertainties.

In this way, unmeasured exclusive fractions were calculated from measured exclusive fractions (the latter were taken from the 2008 Review, and so did not benefit from recent results). In the hadronic sector, the measured total of 59.4% of D_s^+ decays led to an estimated total of 24.2% for unmeasured modes. Weighted counts of π^+, K_S^0, *etc.*, were then made to get the inclusive fractions.

Of interest here is that the sum of all the exclusive fractions—a way-stop in getting the inclusive values—was a nearly correct 103%. In the absence of complete measurements, the model is a way to, in effect, average over ignorance. It probably works better summed over a number of charge-mode sets than in detail. It is known to sometimes give incorrect results when there are sufficient measurements to test it.

References:

1. S. Dobbs *et al.*, Phys. Rev. **D79**, 112008 (2009).
2. P.U.E. Onyisi *et al.*, Phys. Rev. **D88**, 032009 (2013).
3. M. Gronau, J.L. Rosner, Phys. Rev. **D79**, 074022 (2009).